Appendix I A NOTE ON PRIORITIES IN THE CONSTRUCTION OF PHYSICAL SPACE Ted Bastin March 1990

The picture provided by David reminds me a bit of the Summae of Thomas Aquinas. David has simplified his enormous task by making a separation of the material into two. Thomas has the systematic natural theology, which is the domain of reason, and the revealed truth which has to be incorporated in the natural theology but which we can neither justify nor disprove. David's operator calculus is the former, and the hierarchy together with some of program-universe is the latter. There are some puzzling correspondences between the two, and these are my present topic.

I take David's account of dimensionality to get me going because this provides the strangest of these correspondences. I think we need to think out clearly whether (rejecting the revealed truth explanation) the appearance of dimensionality in both types of argument is an accident or coincidence, and, if not, which is the more fundamental appearance of it, and which the consequence. If we reject coincidence, it has to be one way round. David starts by requiring that a correct representation of dimensionality should "use a metric criterion which does not in any way distinguish one dimension from another." He says that "in a continuum theory we would call this the property of "homogeneity and isotropy," though in fact this analogy short-circuits several vital arguments: it is the first statement which is seminal. Historically it was so, for it was *precisely* this argument which started off our whole enterprise. Clive and I (Concept of order I) asked 'what was physical dimensionality?' and concluded that it must be defined as a property within a formal structure in which the mathematical relationships would all remain unchanged as regards truth if the dimensions were interchanged in any possible way. This property we called 'similarity of position.' A weaker condition was called 'simultaneity' in recognition of the fact that if the mathematics permitted us to give any preferential order, then this order could, and would, be used to define temporal relationships. If there were no such order definable then things would be simultaneous. The 'theory-language' embodying this requirement was seen as the simplest level of a hierarchical structure in which we conjectured that the scale-constants would play a part. We had no idea what; though we were clear we had to look elsewhere than Eddington's way of relating levels and calculating the constants. When Frederick solved these problems we found that the 'similarity of position' property was indeed possessed by the level with three dcss however one chose the generators.

For a very long time I was unable to imagine why we had the dimension struc-

ture appearing in the succession of levels as well as the 3-level. I felt sure that the former was the primary case even though that did not fit with any interpretation of the quantum numbers. I only now see that I was still trapped in a classical way of thinking about dimension structure even after all that history, and it took David's presentation at ANPA three (or was it four) years ago to awaken me from my dogmatic nightmares. What I got from him was that it required a recursive structure to define the metrical aspect of dimension even after the combinatorial condition of similarity of position had been provided. The reason is very fundamental, and goes like this: at the basis of the recursive structure of the hierarchy is the idea that we can always collapse our description back down through the levels by grouping sets of elements together and treating them as single elements. Now it must not matter for any assignable mathematical reason which element we finish up at. To put it another way, if there were a substructure in the basis grouping from which we could erect our hierarchy then this would be available to use as a single unit in its own right. This requirement links the recursion with the similarity of position ("isotropy"). It also shows that the recursion collapse must be the combinatorial germ of metrical thinking.

(Scarrott uses a similar argument to show that any concept of *information* capable of introducing meaning - which Shannon/Weaver doesn't - must be recursively structured.)

Now David's current discussion of dimension structure using Feller's result is quite different from all this. Indeed he may repudiate all I say about his thinking. Nevertheless I think both that the connexion which I have been describing is very deep and I got it from David.

I interpolate the comment here that the unification of two meanings for 'dimension' is urgently needed to explain how quantum-number structure can come to have any correspondence with classical ideas of fields, spin and so on. Pierre, in discussion in the autumn was inclined to regard this as a fortunate accident, but I now think we can do better than that. I also point out that in Clive's recent reformulation of the hierarchy structure we are compelled to be flexible about levels; for example the entities in the background have no level defined. This relativism is necessary for my earlier argument, and it was partly the rigidity implied by older understanding of the hierarchy levels that held me up.

David has been a bit hard to pin down about the place he sees the hierarchy occupying in his operator calculus. I think he would like to see it as an *example* of his general and universal scheme. There are a variety of difficulties in that way of thinking (which is why recourse to a revelational role for the hierarchy is tempting.) The Parker-Rhodes cut-off has a certain resemblance to the McGoveran-Feller theorem in that both depend upon rejecting statistically unlikely circumstances. There the resemblance ends. In particular the Feller result requires limiting arguments which it is difficult to give a combinatorial meaning to. I believe Stapp pointed this out, and Pierre and David argued briefly that an alternative combinatorial account could be provided. Unfortunately I cannot remember which document I found that in. By contrast, the Parker-Rhodes cut-off occupies an integral place in Pierre's by now very impressive account of particles derived from scattering and the coupling constants, in the second order approximation.

I hope I have by now said enough to exhibit both the sharpness and the importance of the conflict between the two methods. We have to face up to it. I put forward the following solution for consideration. We take the combinatorial hierarchy account of the origin of dimension structure as the primary one. Then we imagine David asking the question: - is there a statistical treatment of the same problem using something we could plausibly regard as exhibiting an equivalent cutoff, but with a meaning for dimension more like the conventional one? But what is the conventional one? we immediately ask. Here there is scope for invention since there is no classical account of dimensionality which does not depend upon imagined bodily experience. David uses this flexibility in the following way: he adopts the Feller result and the 'isotropy' and then *deduces* the shape that what he calls metric points must take in order to fit in with what he has adopted. The result is his representation of metric space.

This derivation of metric points would have the right form to give Pierre's conservation theorem, though it would now be obvious that it was quite unjustified to drag in the idea of anything being conserved. We notice that we can now use the dimensions as labels for quantum numbers in the restricted sense that they are independent and can be recognized as independent experimentally. Thus we can now say that a motion requires one, two or three labels to specify it, but we can't attach angles. However this is a great step. At this stage we can also introduce the relationship of three-and four-vectors. The four-vector has nothing to do with extending a three-vector by adding another place. (It certainly has nothing to do with a change to a relativistic point of view. I think we are automatically working in a relativistic frame if we accept Pierre's present views on the photon, which seem very satisfactory to me.) The relationship between the vectors is a level change from requiring two strings at level one to having one at level two. By my old argument we get spin into the system by making this change. All these changes have become possible because David has essentially redefined 'dimension.'

Now there is another aspect to the puzzle. David speaks of the metric points as being synchronized, thus referring to his second order correction of the finestructure constant where the number of ways of performing the synchronization gave the correction. Here the number of metric points is indefinite, and it looks as though he is using the same argument as I suggested in defining bound and free states in a note that I wrote not long ago. There I followed Clive in making a distinction between a state in hierarchy construction where one has a completed level and that where one is in the stage of constructing a level. In the latter state one can go on forever. (Clive found it puzzling that one *could* jump into a new level at the first opportunity but never *had* to.) The indefiniteness is equivalent to mapping a geometry onto the dimension structure, and I think we really had now bridged the gap between the combinatorial and the geometrical with David's synchronization as the linking concept.

I go back to Clive's letter of 6/11/89 on conservation at 3- and 4- vertices. He observes that the former cannot conserve both energy and momentum whereas the former [latter ?] must, saying that Pierre would think this too well known to need saying. We might use this as the break-into point for the metrical space by requiring that in going from the combinatorial dimensionality to the metrical one in David's form we -as it were- compensate for the change by treating the combinatorial inexactness (need to impose synchronization) by metrical inexactness which means variable momentum and or energy and or experimental association of angles with counts. (All these things come together and we can't yet describe them separately.)

At this point I ought to start reinterpreting all this in terms of Hamming distances and David's representation of metric points on indefinite strings, but I am going to make a break for first reactions. My whole argument depends on the absolute need to reconcile David's dimensionality theorem with the hierarchy (which is what he mainly uses.) The way I do this is strange and it is crucial that it be right. There are various advantages which come by the way -some more obvious than others. To my mind, the most important is that we have started the job of saying what the quantum numbers are, instead of using the word 'spin' (in particular) and by default saying 'everyone knows what spin means.'

